had obtained, in 1874, the permission of the Académie des Sciences of Paris to consult the original manuscript, but upon searching the archives it was found that it had never been seen after the first printing.

Among my autographs is an interesting document bearing upon the matter and, so far as I have been able to ascertain, thus far unpublished. It consists of four pages in the handwriting of Legendre and is signed by him and Cauchy. On the first page is a note in another hand, "29 Juin 1829." There is nothing to show what this date means, but it seems to indicate that the Académie was considering the printing of the article which had already been presented. At any rate, the report is upon the question of the value of the memoir. It begins with the words: "Report on a memoir relative to a general property of 'une classe très étendue de functions transcendantes.' The Académie has directed M. Legendre[1] and me to report concerning a memoir of M. Abel relative to a general property of a class 'très étendue,' of transcendent functions." It then goes on to describe the nature of the functions and to mention that they have certain properties which are analogous to those of logarithmic and elliptic functions. It speaks of the fact that Abel had already planned for printing a portion of it in Crelle's journal, of his great promise in science, and of the serious loss to the world in his early death, just as he was working upon several new memoirs. It concludes with the recommendation that the memoir be published in the collection of those of foreign scientists.

The recommendation seems to show that it was written about the date mentioned, which was nearly three months after Abel's death, and that this advice was followed. In the publication of 1841, Libri had attempted the necessary corrections.

That there should have seemed to be necessary such a recommendation with respect to a memoir by a man like Abel only goes to show how little he was generally known at the time and how careful the Académie was as to the papers which bore its seal of approval.[2]

QUESTIONS AND DISCUSSIONS.

EDITED BY C. F. GUMMER, Queen's University, Kingston, Ont., Canada.

DISCUSSIONS.

When the forces of modernism have reformed our civil year and reduced its irregularities to a minimum, all calendars will be perpetual, and the Dominical or Sunday letter will cease to control our destinies. Until that time, though less mysterious than those lunar influences that combine with it to generate the movements of the movable feasts, the Sunday letter retains its place in our

[1] Although written by Legendre, he phrases the report as if it was due to Cauchy.

[2] I am indebted to Mr. Jekuthial Ginsburg for aid in tracing some of these facts.

almanacs, and continues to be a source of some trouble and confusion to the unlearned. Cheap printing and advertising methods have today so decorated our walls with calendars that the problem of the perpetual calendar has ceased to be immediate and vital. Nevertheless there is a steady interest in these questions, and some practical importance when remote dates have to be considered. The article by Dr. W. H. Vail appearing below contains a rule for rapidly finding the Dominical letter for any year of our era. Readers should have no difficulty in supplying a proof of the rule. Two other articles on the subject have appeared in this department (*1921*, 127, 260).

As companions to the mnemonic lines cited by Dr. Vail, some others may be noted that were obtained through editorial correspondence. Dr. Edgar Dehn of Waterloo, Canada, contributes this "little verse, sufficiently short not to scare those who don't like verses, yet long enough to help in memorizing" the sequence of digits in the *decimal expansion of* π.

"And I wish I could recollect
3 1 4 1 5 9
My number, known and select."
2 6 5 3 6

There is a recurrent interest in mnemonics for π, but Dr. Dehn was under the impression that such mnemonic curiosities existed only in the French and Russian languages. The Russian verse cannot be printed in the Latin alphabet, and will therefore not be given here. The French lines which follow are well known.[1]

"Que j'aime à faire apprendre un nombre utile aux sages!
3 1 4 1 5 9 2 6 5 3 5
Immortel Archimède, sublime ingénieur,
8 9 7 9
Qui de ton jugement peut sonder la valeur?
3 2 3 8 4 6 2 6
Pour moi ton problème eut de pareils avantages!"
4 3 3 8 3 2 7 9

Numerous variant forms have been mentioned, but few of them compare in elegance with that given above.

The *Scientific American* for March 21, 1914, gave the following:

"See I have a rhyme assisting
My feeble brain, its tasks ofttimes resisting!"

Now it is known that the first cipher in the decimal expansion of π occurs in the thirty-third place. Hence it is pointed out by Professor A. A. Bennett that a mnemonic stanza of exactly thirty-two words serves also to give thirty-three figures, if it be recalled that the digit next in order is a cipher. Professor Bennett cites as a "somewhat inelegant" example of such a complete stanza the following lines, composed by him after seeing the verse in the *Scientific American*.[2]

"Now I read a rhyme, expressed in verses clear,
The ratio circular expanding (science inerudite);
And if old Ludolph's very number do appear,
Then 'tis but Egyptian art, or ancient Syracusan might."

[1] These lines and two in German are quoted in this MONTHLY, *1905*, 215.
[2] Two more examples are given in this MONTHLY, *1906*, 50.

The allusion is to Ludolph van Ceulen (d. 1610), who computed π to 35 places, and in whose honor the term "Ludolph's number" is current in Germany. The references to Egypt and Archimedes of Syracuse need not be discussed.

Mr. Haldeman shows that the vertices of a regular undecagon inscribed in a given circle lie on a certain sextic curve. The equation of the sextic involves a parameter by variation of which the undecagon may be rotated into all positions on the circle. The method is along the same line as one given in the MONTHLY by the same author in connection with the regular pentagon and the regular heptagon (*1920*, 257–258, *1919*, 390). Some editorial remarks follow the article.

Dr. J. Rosenbaum investigates a problem suggested by the Pythagorean relation. It may be shown that for a triangle ABC with $a > b > c$ there exists a unique $n \geqq 1$ such that $a^n = b^n + c^n$; but only when A is a right angle or a straight angle does n depend solely on A. In general there is a relation connecting A, b/c, and n, which becomes independent of b/c when A is put equal to $\pi/2$ or π. Dr. Rosenbaum shows that, conversely, when A and n are given, this relation is satisfied by at most one real value of b/c, except when $n = 1$ or 2.

The note by Professor Swift calls attention to a fact in connection with trigonometric tables which is something of a paradox, and illustrates the distinction between a transcendental number and a transcendental function.

The failure at the origin of the formula in polar coördinates for the direction of a curve appears to receive no notice in a number of calculus texts. Professor H. J. Ettlinger supplies this want in the short note which forms Discussion V. His result could also be obtained by transforming dy/dx into polar form.

I. UNCLE ZADOCK'S RULE FOR OBTAINING THE DOMINICAL LETTER FOR ANY YEAR.[1]

By W. H. VAIL, Newark, N. J.

All of our measurements of time are either natural or conventional in character. Thus the solar year, the solar day, and the lunar month are natural measurements of time, being governed by the movements of the sun, the moon, and the earth in their various orbits; while all the other measurements of time, such as the second, the minute, the hour, the week, the civil month, the civil year, and the century, are conventional or arbitrary in character, having been formed for the convenience of man.

All almanacs and diaries which contain the Church Calendar, and the list of eclipses, with the chronological cycles, will cite among other items the Dominical letter for the year which the diary or almanac represents. Thus for the year 1921 they will state that the Dominical letter is B, which is pure Greek to almost every person who consults it. The first problem which the construction of a calendar presents is to find the day of the week corresponding to a given day of the year. The week contains seven days, and the problem would be perfectly

[1] An explanation of the Dominical Letter with tables is given in *The Calendar, its History, Structure and Improvement*, by Alexander Philip, Cambridge University Press, 1921.